

Quark and pion effective couplings from polarization effects

Fábio L. Braghin

Instituto de Física, Federal University of Goiás,
Av. Esperança, s/n, 74690-900, Goiânia, GO, Brazil

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Abstract

A flavor $SU(2)$ effective model for pions and quarks is derived by considering polarization effects departing from the usual quark-quark effective interaction induced by dressed gluon exchange, i.e. a global color model for QCD. For that, the quark field is decomposed into a component that yields light mesons and the quark-antiquark condensate, being integrated out by means of the auxiliary field method, and another component which yields constituent quarks, which is basically a background quark field. Within a longwavelength and weak quark field expansion (or large quark effective mass expansion) of a quark determinant, the leading terms are found up to the second order in a zero order derivative expansion, by neglecting vector mesons that are considerably heavier than the pion. Pions are considered in the structureless limit and, besides the chiral invariant terms that reproduce previously derived expressions, symmetry breaking terms are also presented. The leading chiral quark-quark effective couplings are also found corresponding to a NJL and a vector-NJL couplings. All the resulting effective coupling constants and parameters are expressed in terms of the current and constituent quark masses and of the coupling g .

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1 Introduction

Hadron and nuclear structure and dynamics are ultimately ruled by Quantum Chromodynamics (QCD) which, due to its intrincated structure, is not exactly solved with currently known analytical methods [1]. There is a large amount of works dedicated to establish connections between (low and intermediary energies) QCD and observable hadrons which rely on the derivation and/or elaboration of effective models, either formally or pheonomenologically [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28]. Hadron effective field models and theories must then be compatible with the more fundamental QCD symmetries and structure. These effective models and theories are expected to describe

observations already in the tree level or with first order corrections and the corresponding effective parameters and coupling constants might be expected to be calculable from QCD grounds. Several effective models for low energy hadron structure and interactions are formulated in terms of pions and constituent quarks and gluons, which have shown to be a powerful way to describe many hadron properties [24, 25]. Currently there is a large effort by many groups to describe nuclear properties by considering hadron effective models or theories, few examples are given in [5, 11, 16, 22, 23, 24, 25, 28, 29, 30], whose QCD content is more transparent. Dynamical chiral symmetry breaking (D χ SB) is an extremely important QCD effect to be taken into account since it yields broad and well known consequences in the hadron level and a direct relation to confinement is expected, for example in [1, 31, 32]. The relevance of the corresponding chiral condensate for hadron structure and dynamics is widely recognized although recently its location became a controversial subject [33, 34]. When establishing relations between QCD and hadron phenomenology it is of utmost importance to understand each piece of the mechanisms and effects that provides the measurable quantities. The fundamental effects that yield hadron effective interactions/couplings might help to select the most realistic and relevant couplings in a hadron model. One of the most investigated sources of quark effective couplings is instanton mediation [35, 36, 37, 12] and polarization effects have also been shown to produce quark effective interactions [26, 27].

D χ SB is to be investigated and understood in the QCD quark sector and although this program still needs theoretical developments and improvements, one might address particular known limits of the full theory. The following low energy quark effective global color model [6, 3, 4] will be considered:

$$Z = N \int \mathcal{D}[\bar{\psi}, \psi] \exp i \int_x [\bar{\psi} (i\not{\partial} - m) \psi - \frac{g^2}{2} \int_y j_\mu^b(x) (\tilde{R}_{bc}^{\mu\nu})(x-y) j_\nu^c(y) + \bar{\psi} J + J^* \psi]$$

Where the color quark current is $j_a^\mu = \bar{\psi} \lambda_a \gamma^\mu \psi$, the sums in color, flavor and Dirac indices are implicit, \int_x stands for $\int d^4x$, the kernel $\tilde{R}_{bc}^{\mu\nu}$ is a dressed gluon propagator. Even if other terms might arise from the non abelian structure of the gluon sector, the quark-quark induced interaction (1) should be part of the quark effective action for QCD. Although simple one gluon exchange is known to not produce enough strength of quark interactions such as to yield D χ SB there are different effects that improve such picture and make the strength strong enough. For example by considering gluons self interactions in the polarization tensor, by modeling confinement or corrections in the corresponding Schwinger Dyson approach already in the rainbow ladder approximation, few examples are given in Refs. [7, 38, 39, 40].

The aim of this work is to derive a flavor SU(2) low energy effective model when internal structure of pions is not important by departing from the generating functional for the model (1). A quark field splitting into two components is performed: one component whose composite excitations correspond to light quark-antiquark mesons and the scalar condensate, and the other that remains as (constituent) background field quarks. If these components may correspond or not to low or high energy quark components will not be discussed since this procedure corresponds to the one-loop background field method [41, 42]. Vector mesons degrees of freedom

will be neglected since they are considerably heavier than the pion and are not expected to contribute in the low energy limit. The analysis of problems related to the light vector mesons is left for another work. The quark and pion effective couplings emerge from an expansion of a quark determinant which therefore generates a series of interactions with progressively higher powers of quark bilinears and of the pion field and their derivatives. Mesons will be considered in the punctual approximation within the zero order derivative expansion and this basically reproduces the corresponding terms found previously in Refs. [6, 4] in the chiral limit. In addition to that, chiral symmetry breaking terms due to the current quark mass are also found in the punctual pion limit. Besides that, leading quark-quark effective couplings, such as the NJL coupling, are also exhibited, being higher order in $1/N_c$ in agreement with Ref. [43]. We also believe this approach might provide insights for the investigation of the longstanding problem of the convergence of the QCD effective action. An effective field theory (EFT) for low and intermediary energies QCD might be though as enough to provide a reliable understanding of the corresponding processes with enough predictive power to describe Nature at this level. However, this picture becomes more complete if the more fundamental mechanisms that generate all the terms of the corresponding EFT, with their effective parameters, are found or derived. With the present work we hope to provide further insight into this program by considering the global color model (1).

The paper is organized in the following way. In the next section, with a Fierz transformed version of the above non local current-current quark effective interaction, the quark field is separated into two components. One of these components will be integrated out by considering a set of auxiliary fields which generate quark-antiquark meson fields and the chiral condensate. The other component is dressed by polarization effects. Auxiliary fields that correspond to vector mesons will be neglected since they are considerably heavier than pions and do not contribute in the low energy regime. With a chiral rotation in Sect. 2.2, the non linear realization of chiral symmetry is introduced in terms of covariant derivatives. The expansion of the quark determinant is exhibited in Section 3, where the effective coefficients of the leading terms of quark couplings are presented up to the second order and the pion sector up to the fourth order. In the last Section there is a summary and discussion.

2 Flavor structure and auxiliary fields

The departure point is therefore expression (1), and the kernel $\tilde{R}_{ab}^{\mu\nu}$ can be written in terms of transversal and longitudinal components such that:

$$\tilde{R}_{ab}^{\mu\nu} = \delta_{ab} \left[R_T \left(g^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{\partial^2} \right) + R_L \frac{\partial^\mu \partial^\nu}{\partial^2} \right]. \quad (1)$$

With a Fierz transformation [6, 3, 28] by selecting only the color singlet terms, the quark interaction above can be written in terms of bilocal quark bilinears, $j_i^q(x, y) = \bar{\psi}(x) \Gamma^q \psi(y)$ where $q = s, p, v, a$ and $\Gamma_s = I_2 \cdot I_4$ (for the 2x2 flavor and 4x4 identities), $\Gamma_p = i\gamma_5 \sigma_i$, $\Gamma_v^\mu = \gamma^\mu \sigma_i$ and $\Gamma_a^\mu = i\gamma_5 \gamma^\mu \sigma_i$, where σ_i are the flavor SU(2) Pauli matrices. The resulting non local

interactions are the following:

$$\begin{aligned}
\Omega &\equiv g^2 j_\mu^a(x) \tilde{R}_{ab}^{\mu\nu}(x-y) j_\nu^b(y) \\
&\rightarrow \alpha g^2 \left\{ [j_S(x, y) j_S(y, x) + j_P^i(x, y) j_P^i(y, x)] R(x-y) \right. \\
&\quad \left. - \frac{1}{2} [j_\mu^i(x, y) j_\nu^i(y, x) + j_{\mu A}^i(x, y) j_{\nu A}^i(y, x)] \bar{R}^{\mu\nu}(x-y) \right\}, \tag{2}
\end{aligned}$$

where $i, j, k = 0, \dots, (N_f^2 - 1)$ and $\alpha = 8/9$ for SU(2) flavor. The kernels above can be written as:

$$\begin{aligned}
R(x-y) &\equiv R = 3R_T + R_L, \\
\bar{R}^{\mu\nu}(x-y) &= g^{\mu\nu}(R_T + R_L) + 2 \frac{\partial^\mu \partial^\nu}{\partial^2} (R_T - R_L). \tag{3}
\end{aligned}$$

The local limit yields the Nambu Jona Lasinio (NJL) and vector NJL couplings. The coupling constants are roughly, for massless gluons, $G \sim \frac{g^2}{\Lambda_{QCD}^2}$ [44] or $G \sim \frac{g^2}{M_G^2}$ for non zero effective gluon mass being comparable in any case [26, 28, 45, 46, 47, 48]. These couplings are of the order of $G \sim g^2 \sim 1/N_c$.

The quark field will be splitted such as to preserve chiral symmetry into a component, $(\psi)_2$, that yields the scalar condensate and whose (composite) excitations correspond to (quark-antiquark) light mesons, and another component, $(\psi)_1$, that will be associated to constituent quark. If the usual shift were performed with a background fermion field, for $\psi \rightarrow \psi + \psi_1$ and $\bar{\psi} \rightarrow \bar{\psi} + \bar{\psi}_1$, additional contributions of higher order would emerge. This field separation by means of the bilinears yields rather the background field method in the one-loop level [41, 42]. The shift in the bilinears also produces automatically chiral invariant structures, being however that quarks that are integrated out also provide bound light meson states and the quark-antiquark condensate. Physically, this shift of bilinears can also be associated to the quark-antiquark states built with auxiliary fields. Quark bilinears will therefore be written as:

$$\bar{\psi} \Gamma^a \psi \rightarrow t_2 (\bar{\psi} \Gamma^a \psi)_2 + t_1 (\bar{\psi} \Gamma^a \psi)_1, \tag{4}$$

where t_1 and t_2 are constants that can be set to one at the end and that help to understand the role of each of these components in the resulting model. According to this separation, the current-current quark interactions can be written in three parts: $\Omega \rightarrow t_1^2 \Omega_1 + t_2^2 \Omega_2 + t_1 t_2 \Omega_{12}$ where Ω_{12} mixes both components and Ω_1, Ω_2 stand for the terms exclusive to each of the components. Ω_{12} can be written as:

$$\begin{aligned}
\Omega_{12} &\equiv \alpha g^2 [j_{i1}^S(x, y) R j_{i2}^S(x, y) + j_{i1}^P(x, y) R j_{i2}^P(x, y) \\
&\quad + j_{i2}^S(x, y) R j_{i1}^S(x, y) + j_{i2}^P(x, y) R j_{i1}^P(x, y)] \\
&\quad - \frac{\alpha g^2}{2} \bar{R}_{\mu\nu} [j_{i1}^\mu(x, y) j_{i2}^\nu(x, y) + j_{iA1}^\mu(x, y) j_{iA2}^\nu(x, y) \\
&\quad + j_{i2}^\mu(x, y) j_{i1}^\nu(x, y) + j_{iA2}^\mu(x, y) j_{iA1}^\nu(x, y)].
\end{aligned}$$

The resulting ambiguity in this splitting will not be solved here. However it will be shown below that it yields the ambiguity of determining the relative contribution of constituent quarks and pions (or pion cloud) to describe hadron observables, in particular baryons [5, 49].

2.1 Auxiliary fields and pions

To integrate out the component $(\bar{\psi}\psi)_2$, a set of bilocal auxiliary fields (a.f.) with the quantum numbers of the bilinears defined above is introduced to linearize Ω_2 . The generating functional is multiplied by a collection of normalized unity Gaussian integrals of the a.f., and these fields are shifted by fermion bilinears, preserving an unit Jacobian, such that all the terms in Ω_2 are canceled out. These integrals, with the corresponding shifts, are given by:

$$\begin{aligned}
1 = & N \int D[S]D[P_i] e^{-\frac{i}{2}t_2^2 \int_{x,y} R\alpha[(S-gj_{(2)}^S)^2+(P_i-gj_{i,(2)}^P)^2]} \\
& \int D[V_\mu^i] e^{-\frac{i}{4}t_2^2 \int_{x,y} \bar{R}^{\mu\nu}\alpha[(V_\mu^i-gj_\mu^{i,(2)}) (V_\nu^i-gj_\nu^{i,(2)})]} \\
& \int D[\bar{A}_\mu^i] e^{-\frac{i}{4}t_2^2 \int_{x,y} \bar{R}^{\mu\nu}\alpha[(\bar{A}_\mu^i-gj_\mu^{i,(2)A}) (\bar{A}_\nu^i-gj_\nu^{i,(2)A})]}.
\end{aligned} \tag{5}$$

The bilocal a.f. are $S(x, y), P_i(x, y), V_\mu^i(x, y)$ and $\bar{A}_\mu^i(x, y)$ and the corresponding shifts (with unity Jacobian) were given by the following bilocal bilinears:

$$\begin{aligned}
j_S &= \bar{\psi}(x)\psi(y), & j_P^i &= \bar{\psi}(x)i\gamma_5\sigma_i\psi(y), \\
j_\mu^i &= \bar{\psi}(x)\sigma_i\gamma_\mu\psi(y), & j_\mu^{i,A} &= \bar{\psi}(x)\sigma_i i\gamma_5\gamma_\mu\psi(y).
\end{aligned} \tag{6}$$

The resulting effective Lagrangian is given by:

$$\begin{aligned}
\mathcal{L}_1 = & \int_y t_2 \bar{\psi}_2 \left[(i\gamma \cdot \partial - m)\delta_{x,y} + t_2 \alpha g R(S + i\gamma_5\sigma_i P_i) + \frac{t_2}{2} \alpha g \bar{R}^{\mu\nu} (V_\mu^i \gamma_\nu + \bar{A}_\mu^i i\gamma_5 \gamma_\nu) \right. \\
& \left. + t_1 \alpha g^2 \sum_q R^q(x, y) \Gamma_q j^q(x, y) \right] \psi_2 + t_1 \bar{\psi}_1 (i\gamma \cdot \partial - m) \psi_1 \\
& - \frac{g^2 t_1^2}{2} \int_y j_\mu^{b,(1)}(x) \tilde{R}_{bc}^{\mu\nu}(x, y) j_\nu^{c,(1)}(y) - \frac{t_2^2 \alpha}{2} \int_y \left\{ R[S^2 + P_i^2] + \frac{1}{2} \bar{R}^{\mu\nu} [V_\mu^i V_\nu^i + \bar{A}_\mu^i \bar{A}_\nu^i] \right\}
\end{aligned} \tag{7}$$

where terms of the form $R^q(x, y) \Gamma_q j^q(x, y)$ correspond to the terms from Ω_{12} , being $R^q = (R, \bar{R}^{\mu\nu})$ with the corresponding operators Γ^q from the bilinears. In this expression the inverse Fierz transformation was performed for the terms in $t_1^2 \Omega_1$ which was written as a current-current effective interaction again. By integrating out the quark field ψ_2 , in the limit of zero quark field ψ_1 (or $t_1 = 0$) and zero quark mass, the resulting model is the same as the model presented and investigated in Refs. [2, 4, 6] in the flavor SU(3) version. Therefore the resulting pion sector has the same structure. It has also been shown that a chiral rotation in the measure of the generating functional yields a Wess Zumino term [4, 6]. Since the pion sector obtained in these works is the same as the one obtained in the present article the calculation will not be exhibited here.

From this non local theory, the local meson fields are defined by means of a formal expansion of the bilocal a.f. $\phi_q(x, y)$ on a local meson field basis $M_{k,q}$ which is given by:

$$\phi_q(x, y) = \tilde{F}_q(x - y) + \sum_k M_{k,q} \left(\frac{x + y}{2} \right) F_{k,q}(x - y), \tag{8}$$

where $F_{k,q}$ are the form factors associated to the corresponding $k = 0, 1, 2, \dots$ -meson excitation of the channel q . \tilde{F}_q (for $q = s, p, v, a$) correspond to the translational invariant vacuum functions for each of the channels q and $M_{k,q}$ are the local meson fields, being that $k > 0$ correspond to all meson excitations in the corresponding quark-antiquark channel q . The aim of the present work is to obtain a local meson-quark effective model for the low energy regime in which internal structure of mesons is not relevant. Therefore only the local limit of these form factors will be considered, and it will be written as: $F_{k,q}(x-y)R^q(y-x) \simeq F_{k,q}\delta(x-y)$. This limit yields punctual mesons. Besides that, only the lowest quark-antiquark scalar and pseudoscalar states (mesons) should contribute, i.e. $k = 0$ for the local meson fields denoted by $M_{k=0,q} = s, p_i, v_\mu^i, a_\mu^i$ since higher quark-antiquark excited states are heavier and only contribute for (relatively) higher energy processes. Finally, the vector/axial (quark-antiquark) mesons do not contribute for low energy regime since their masses are considerably higher than the pion mass. Even if the auxiliary fields for vector mesons were considered, their structure and dynamics could receive contributions from constituent quarks and pion cloud, inducing an ambiguity in their description similar to the one that will be found for baryons. This problem however is outside the scope of this work. Moreover, vector mesons are known to give rise to corrections for the Skyrme terms (fourth order pion couplings c_1, c_2 found below) [50] and therefore to some extent their contribution can be incorporated by redefining the fourth order pion terms. Chiral transformations mix scalar and pseudoscalar fields and therefore, in the limit of small current quark masses, one must have $F_{0,s} = F_{0,ps} = F$. From here on, only the local punctual meson fields leading terms will be considered. This will produce the correct punctual meson limit of the previously derived low energy pion effective couplings [6, 3, 4, 12].

Consider the following terms from the quark and auxiliary fields interaction, $\bar{\psi}_2(x)\Xi(x,y)\psi_2(y)$ where:

$$\begin{aligned} \Xi(x,y) = & g\alpha \{ F_{0,0}(x-y)R[S(z) + P_i(z)i\gamma_5\sigma_i] \\ & - \frac{\gamma_\nu\sigma_i}{2}\bar{R}^{\mu\nu} [F_0^v(x-y)V_\mu^i(z) + i\gamma_5 F_0^a(x-y)\bar{A}_\mu^i(z)] \} \end{aligned} \quad (9)$$

where $z = \frac{x+y}{2}$ that reduces to $z = x$ due to the structureless mesons approximation, and, in the absence of the heavier vector mesons, it reduces to:

$$\begin{aligned} \Xi(x,y) & \rightarrow \Phi_L(x,y) \simeq F(s + p_i\gamma_5\sigma_i)\delta(x-y) \\ & \equiv \tilde{\Phi}_L\delta(x-y). \end{aligned} \quad (10)$$

Therefore, the quark ψ_2 and meson coupling in terms of $\Xi \sim \tilde{\Phi}\delta^4(x-y)$ can be written as: $t_2^2 \bar{\psi}_2\Phi_L(x-y)\psi_2$, which corresponds to the linear realization of chiral symmetry. The canonically normalized definition of the pion field becomes: $\vec{\xi} = F\vec{p}$.

2.2 Chiral rotation

The non linear representation for chiral symmetry can be obtained as described below. The scalar field is frozen and then, by performing a chiral rotation, only the pion field and its (covariant) derivative remain [51, 52, 42]. This can be done by constraining the scalar and

pseudoscalar fields to the chiral radius, $1 = s^2 + \vec{p}^2$ which yields $s = \sqrt{1 - \vec{p}^2}$. Now we note there is a freedom to define the pion and quark fields and derivatives related among each other by chiral rotations. The quark free terms and its coupling to (scalar and pseudoscalar) mesons are given by:

$$\begin{aligned} & t_2 \bar{\psi}_2 [i\gamma \cdot \partial - m + t_2 \Phi_L] \psi_2 \\ & = t_2 \bar{\psi}_2 [i\gamma \cdot \partial - m + t_2 F(s + i\gamma_5 \vec{\sigma} \cdot \vec{p})] \psi_2, \end{aligned} \quad (11)$$

Quark and scalar and pseudoscalar fields can be redefined as [51, 52, 42]:

$$s = \frac{1 - \vec{\pi}^2}{1 + \vec{\pi}^2}, \quad p_i = \frac{2\pi_i}{1 + \vec{\pi}^2}, \quad \psi = \frac{(1 - i\gamma_5 \vec{\sigma} \cdot \vec{\pi})}{\sqrt{1 + \vec{\pi}^2}} \psi'. \quad (12)$$

In the resulting non linear realization of chiral symmetry the above Lagrangian terms (11) can be written as:

$$\begin{aligned} & \bar{\psi}'_2 \left[i\gamma \cdot \partial - m^* + \gamma^\mu \vec{\sigma} \cdot \left(\frac{\partial_\mu \vec{\pi}}{1 + \vec{\pi}^2} i\gamma_5 + i \frac{\vec{\pi} \times \partial_\mu \vec{\pi}}{1 + \vec{\pi}^2} \right) \right. \\ & \quad \left. + 4m \left(\frac{\vec{\pi}^2}{1 + \vec{\pi}^2} - \frac{\epsilon_{ijk} \sigma_k \pi_i \pi_j}{1 + \vec{\pi}^2} \right) \right] \psi'_2, \end{aligned} \quad (13)$$

where it was used that $\sigma_i \sigma_j = \delta_{ij} + i\epsilon_{ijk} \sigma_k$, and it has been set

$$t_2 = 1.$$

The last two terms in this expression correspond to the chiral symmetry breaking term from the current quark mass. These terms yield the terms proportional to the pion mass or to powers of $\vec{\pi}^2$ in the resulting effective model. To improve the notation two covariant derivatives are defined as:

$$\begin{aligned} & \mathcal{D}_\mu \vec{\pi} \equiv \frac{\partial_\mu \vec{\pi}}{(1 + \vec{\pi}^2)}, \\ & \bar{\psi}_2 \partial_\mu \psi_2 \rightarrow \bar{\psi}'_2 D_\mu^c \psi'_2 \equiv \bar{\psi}'_2 \left(\partial_\mu + i\vec{\sigma} \cdot \frac{\vec{\pi} \times \partial_\mu \vec{\pi}}{1 + \vec{\pi}^2} \right) \psi'_2. \end{aligned} \quad (14)$$

The canonically normalized pion field corresponds to $\vec{\pi}' = \vec{\pi} F$. From here on, the new definitions of pion and quark field will be used by writing simply π_i and ψ_2 respectively. This redefinition of the fields however induces a non trivial change in the functional measure with terms that do not depend on this pion covariant derivative. These terms are of higher order in the pion and quark fields, therefore they should be less important from a dynamical point of view. This subject will not be addressed further in the present work, and therefore the Jacobian will not be exhibited and discussed here.

A different parameterization of the non linear realization can be used for the pseudoscalar fluctuations around the vacuum to rewrite expression (11), as discussed in Refs. [4, 6], by means of :

$$\Phi_L \rightarrow \Phi_{NL} = F (P_R U + P_L U^\dagger), \quad (15)$$

where $U = e^{i\vec{\sigma} \cdot \vec{\pi}}$ and $P_{R,L} = (1 \pm \gamma_5)/2$ are the chirality projectors. These expressions allow to rewrite the pion sector in the standard shape of Chiral Perturbation Theory.

2.3 Integrating out quarks

By integrating out the component $(\bar{\psi}\psi)_2$ the following non linear (non local) effective action for quarks $(\bar{\psi}\psi)_1$ and pions is obtained:

$$S_{eff} = -i \text{Tr} \log \left\{ (S_0^c)^{-1} + \Phi_N - \alpha t_1 g^2 \bar{R}^{\mu\nu} \gamma_\mu \sigma_i \left[(\bar{\psi} \gamma_\nu \sigma_i \psi)_1 + i \gamma_5 (\bar{\psi} i \gamma_5 \gamma_\nu \sigma_i \psi)_1 \right] \right. \\ \left. + 2 \alpha t_1 g^2 R \left[(\bar{\psi} \psi)_1 + i \gamma_5 \sigma_i (\bar{\psi} i \gamma_5 \sigma_i \psi)_1 \right] + 4m \frac{(\vec{\pi}^2 - \epsilon_{ijk} \sigma_k \pi_i \pi_j)}{1 + \vec{\pi}^2} \right\} - \int \mathcal{L}_2, \quad (16)$$

where the following relation was used: $\det(A) = e^{Tr \ln A}$ and where Tr stands for traces of discrete internal indices and integration of spacetime or momentum coordinates for the quark component ψ_2 . The following kernel has been defined:

$$(S_0^c)^{-1} \equiv (i \gamma_\mu \cdot D_c^\mu - m^*). \quad (17)$$

The contribution of the pion covariant derivative was written as:

$$\Phi_N = i \gamma_5 \gamma^\mu \vec{\sigma} \cdot \mathcal{D}_\mu \vec{\pi} \quad (18)$$

and the remaining terms for the first component of the quark field given by:

$$\mathcal{L}_2 = t_1 \bar{\psi}_1 (i \gamma \cdot \partial - m) \psi_1 - \frac{g^2 t_1^2}{2} \int_y j_\mu^{b,(1)}(x) \tilde{R}_{bc}^{\mu\nu}(x, y) j_\nu^{c,(1)}(y), \quad (19)$$

The a.f. vacuum expected values can be found from their gap equations. However these equations must be found from the effective action in terms of the fields s, p_i , i.e. by integrating out quarks ψ_2 without doing the chiral rotations of the last section. These saddle point equations correspond to:

$$\left. \frac{\partial S_{eff}}{\partial \phi_i} \right|_{[\phi_j^{(0)} = s^{(0)}, p_i^{(0)}, \dots]} = 0. \quad (20)$$

This set of equations corresponds basically to the usual set of gap equations of the NJL model being that in the Nambu Goldstone mode only the scalar field has a non zero expected value in the vacuum. This provides the only contribution to the quark effective mass that constitutes pions and that condense into the chiral condensate, $m^* = m + g s^{(0)}$. These gap equations were solved, for example, in a model with a very simplified gluon propagator in Ref. [27] in terms of an ultraviolet Euclidean cutoff. When a particular gauge is chosen for R and \bar{R} , the gauge fixing parameter can be determined by a condition of gauge independence such as: $\frac{\partial S_{eff}}{\partial \lambda} = 0$. All the quantities in the effective action found below for quarks and pions, and also the gap equations above, depend basically on the original QCD Lagrangian parameters: u-d current quark masses, gauge coupling g , a gauge fixing parameter λ .

By factorizing $\log((S_0^c)^{-1})$ in the determinant, with the quantity $\chi = \gamma^\mu \vec{\sigma} \cdot \frac{\vec{\pi} \times \partial_\mu \vec{\pi}}{1 + \vec{\pi}^2}$, this term can be written as:

$$S'_{d2} = -i \text{Tr} \log [i \gamma \cdot \partial - m^* - \chi] = C_0 + S_{d2} \\ = -i \text{Tr} \log [i \gamma \cdot \partial - m^*] - i \text{Tr} \log [1 - S_0 \chi], \quad (21)$$

where $S_0 = S_0^c(\pi_i = 0)$, being that the first term (C_0) becomes a multiplicative constant factor in the generating functional and the second one can be expanded for weak pion field or in a longwavelength expansion. The first order term is zero, and the expansion can be written as:

$$S_{d2} = i \sum_n \frac{1}{n} \text{Tr} (-S_0 \chi)^n. \quad (22)$$

The expansion of S_{d2} yields terms of higher order in the pion field than the expansion of the pion sector of the remaining part of the determinant. The quark determinant can then be written as:

$$S_{det} = S_d + S_{d2},$$

where the main part can now be written as:

$$S_d = -i \text{Tr} \log \left[1 + S_0^c \left(\tilde{\Phi}_N + 4m \frac{(\vec{\pi}^2 - \epsilon_{ijk} \sigma_k \pi_i \pi_j)}{1 + \vec{\pi}^2} + g^2 \alpha t_1 \sum_q R_q \Gamma_q \bar{\psi} \Gamma_q \psi \right) \right], \quad (23)$$

The pion coupling in D_μ^c also produces further interactions in the expansion as shown below, however the most relevant one is the first order term. This determinant will be expanded in the longwavelength limit (low pion momenta) and for weak ψ_1 quark field (or for small g^2). This expansion is also equivalent to a large quark mass (m^*) zero order derivative expansion [53].

3 Effective quark and pion couplings

By neglecting vector meson fields the expanded determinant can be written as:

$$\begin{aligned} S_d \simeq & i \text{Tr} \sum_n c_n \left\{ S_0 [\Phi_N + 2K_0 R(x-y) [(\bar{\psi}(x)\psi(y)) + \gamma_5 \sigma_i (\bar{\psi}(x) \gamma_5 \sigma_i \psi(y))] \right. \\ & - K_0 \bar{R}^{\mu\nu}(x-y) \gamma_\mu \sigma_i [\bar{\psi}(x) \gamma_\nu \sigma_i \psi(y) + i \gamma_5 \bar{\psi}(x) i \gamma_5 \gamma_\nu \sigma_i \psi(y)] \\ & \left. + 4m \frac{\vec{\pi}^2 - \vec{\sigma} \cdot \vec{\pi} \times \vec{\pi}}{1 + \vec{\pi}^2} \right\}^n \end{aligned} \quad (24)$$

where: $c_n = \frac{(-1)^{n+1}}{n}$, and $K_0 = \alpha g^2 t_1$. All the terms of this expansion will be calculated in the zero order derivative expansion. Besides that, only the leading terms in the pion derivative will be shown, i.e., terms of higher order in $(\partial^n \vec{\pi})$ ($n \geq 2$) will be neglected.

Many terms in this expansion are zero due to the traces of Dirac and Pauli matrices. The only non zero first order terms yield a correction to the quark $(\bar{\psi}\psi)_1$ mass and a pion mass term. They are respectively the following:

$$\mathcal{L}_1 = t_1 \Delta m^* (\bar{\psi}\psi)_1 - M_\pi^2 F^2 \frac{\vec{\pi}^2}{1 + \vec{\pi}^2}, \quad (25)$$

and where these masses were defined as:

$$\Delta m^* = -i N_c \alpha g^2 \text{Tr}' S_0 R, \quad (26)$$

$$M_\pi^2 = i N_c \frac{1}{F} \text{Tr}' S_0 m, \quad (27)$$

where Tr' , from here on, corresponds to traces in all internal and spacetime (or momentum) indices except color. With the help of the gap equation for the scalar field (20), it can be noticed that expression (27) corresponds to the Gell Mann Oakes Renner relation: $M_\pi^2 F^2 = -\langle \bar{q}q \rangle / m_q$. In this expression it is seen that pion mass becomes zero in the chiral limit ($m_q = 0$), i.e. the Goldstone theorem is satisfied. i.e. in the limit of structureless pions. If pion structure had not been neglected, expression above with the corresponding gap equation for the quark (and eventually gluon) propagator yield a rainbow ladder Schwinger Dyson and Bethe Salpeter equations which had been solved previously [54, 55] and that is outside the scope of this work.

3.1 Manohar and Georgi expansion

By neglecting any coupling to gluons the series above yields terms of the following general type [22] for $C = 0$:

$$I_{ABCD} \sim \mathcal{C}_{A,B}^{C,D} \left(\frac{\pi}{f}\right)^A \left(\frac{\bar{\psi}\Gamma\psi}{f^2\Lambda}\right)^B \left(\frac{\mathcal{G}[A_\mu^{1,a}]}{\Lambda}\right)^C \left(\frac{p}{\Lambda}\right)^D \quad (28)$$

which was considered by Manohar and Georgi where $\mathcal{C}_{A,B}^{C,D}$ are the coefficients to be calculated, $(\mathcal{G}[A_\mu^{a,(1)}])^C$ are gauge invariant combinations of A_μ^a which are not considered in this work ($C = 0$), with the chiral invariant combinations of interacting mesons/pions or quarks (A, B) with momenta of order D . Basically the $n - th$ term in the expansion above corresponds to $A + B = n$. Momentum dependence will be considered solely for the pion field in this work.

3.2 Second order terms

There is a first order term of the expansion above that yields a second order pion-quark coupling if the kernel S_0^c is also expanded for the pion coupling χ in the first order, i.e. similarly to the expansion (22). The resulting term is the same as the one emerging from the chiral rotation for the component ψ_2 by considering the zero order derivative expansion. It can be written as:

$$\mathcal{L}_{2d\pi} = i t_1 g_{\pi d\pi} \frac{\vec{\pi} \times \partial^\nu \vec{\pi}}{1 + \vec{\pi}^2} \cdot (\bar{\psi} \gamma_\nu \vec{\sigma} \psi), \quad (29)$$

where it was defined the following coupling constant:

$$g_{\pi d\pi} g_{\rho\nu} \delta_{ij} = i g^2 \alpha N_c Tr' S_0^2 \sigma_i \gamma_\rho \bar{R}_{\mu\nu} \gamma^\mu \sigma_j. \quad (30)$$

In expression (30) the following trace properties must be used:

$$\begin{aligned} tr(\sigma_i \sigma_j) &= 2\delta_{ij}, \\ tr(\gamma_\mu \gamma_\nu) &= 4g_{\mu\nu}, \\ tr(\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma) &= 4(g_{\mu\nu} g_{\rho\sigma} + g_{\mu\sigma} g_{\nu\rho} - g_{\mu\rho} g_{\nu\sigma}), \end{aligned} \quad (31)$$

and, besides that, rotational invariance for the traces in spacetime or momentum coordinates. These properties must be used in the second order terms of the expansion, in particular those

for the Lorentz indices and rotational invariance. If $g_{\pi d\pi} = 1$ this coupling is precisely that coupling from the chiral rotation in the quark covariant derivative (14).

The other second order terms are those for free meson terms, quark-meson and quark-quark effective interactions and they are presented below in the zero order derivative expansion. The resulting pseudoscalar kinetic term is given by:

$$\Delta\mathcal{L}_{freemesons} = \frac{k_2}{2} \mathcal{D}_\mu \pi_i \mathcal{D}^\mu \pi_i, \quad (32)$$

where

$$k_2 g_{\mu\nu} \delta_{ij} = i N_c \text{Tr}' \sigma_i \sigma_j \gamma_5 \gamma_\mu \gamma_5 \gamma_\nu S_0^2. \quad (33)$$

This expression corresponds to the punctual pion limit of the complete expression investigated in Refs. [6, 3, 4], being $k_2 = f_\pi^2$.

The leading quark-meson coupling is the axial coupling [17, 56, 57, 58] that is $\mathcal{O}(N_c^0)$. This term is given by:

$$\mathcal{L}_2^I = -2g_A t_1 (\mathcal{D}_\mu \pi_i) \bar{\psi} i \gamma_5 \gamma^\mu \sigma_i \psi, \quad (34)$$

Where the axial coupling constant was defined as:

$$g_A \delta_{ij} g_{\rho\mu} = -\frac{i}{2} \alpha g^2 N_c \text{Tr}' [\gamma_\rho \gamma_\mu (-1) \gamma_5^2 \sigma_i \sigma_j \bar{R} S_0^2], \quad (35)$$

where $\bar{R} = g_{\mu\nu} \bar{R}^{\mu\nu}$ and the factor (-1) is due to the anticommutation of Dirac matrices. This coupling constant depends strongly on the gluon propagator, and all the effective quark couplings will have this dependence.

Also, there is a symmetry breaking pion-quark term in the second order given by:

$$\mathcal{L}_2^{sb} = t_1 c_{\sigma, sb} \frac{\vec{\pi}^2}{1 + \vec{\pi}^2} (\bar{\psi} \psi) \quad (36)$$

where the coefficient is given by:

$$c_{\sigma, sb} = i 4g^2 \alpha N_c \text{Tr}' m S_0^2 R. \quad (37)$$

Expression (36) is the sigma term interaction.

3.3 Second order quark effective interactions

The longwavelength limit of the leading chiral quark-quark couplings contains coupling constants which are of the order of $1/N_c$. The corresponding second order terms of the determinant expansion above are given by:

$$\begin{aligned} \mathcal{L}_{4q} &= t_1^2 g_{NJL}^{h.o.} [(\bar{\psi} \psi)^2 + (\bar{\psi} \sigma_i \gamma_5 \psi)^2] \\ &+ t_1^2 g_{vNJL} [(\bar{\psi} \sigma_i \gamma_\mu \psi)^2 + (\bar{\psi} \sigma_i \gamma_\mu \gamma_5 \psi)^2]. \end{aligned} \quad (38)$$

For these expressions, the coupling constants were defined within the zero order derivative expansion in the following way:

$$\begin{aligned}
g_{NJL}^{h.o.} &\rightarrow -2ig^4 t_1^2 \alpha^2 \text{Tr} S_0^2 R^2 \\
&\quad \times [(\bar{\psi}\psi)^2 - \sigma_i \sigma_j i^2 \gamma_5^2 (\bar{\psi} i \gamma_5 \sigma_j \psi)(\bar{\psi} i \gamma_5 \sigma_i \psi)] \\
&= t_1^2 g_{NJL}^{h.o.} [(\bar{\psi}\psi)^2 + (\bar{\psi} \gamma_5 \sigma_j \psi)^2] \\
g_{vNJL} &\rightarrow -\frac{i}{2} g^4 t_1^2 \alpha^2 \text{Tr} \bar{R}^{\mu\nu} \bar{R}^{\rho\sigma} S_0^2 \sigma_i \sigma_j \gamma_\mu \gamma_\rho \\
&\quad \times [(\bar{\psi} \sigma_i \gamma_\nu \psi)(\bar{\psi} \sigma_j \gamma_\sigma \psi) - i^2 \gamma_5^2 (\bar{\psi} i \gamma_5 \sigma_i \gamma_\nu \psi)(\bar{\psi} i \gamma_5 \sigma_j \gamma_\sigma \psi)] \\
&= t_1^2 g_{vNJL} [(\bar{\psi} \sigma_i \gamma_\nu \psi)^2 + (\bar{\psi} \gamma_5 \sigma_i \gamma_\nu \psi)^2],
\end{aligned} \tag{39}$$

where:

$$\begin{aligned}
g_{NJL}^{h.o.} &= -2ig^4 \alpha^2 N_c \text{Tr}' S_0^2 R^2, \\
g_{vNJL} g_{\mu\rho} \delta_{ij} &= -\frac{i}{2} g^4 \alpha^2 N_c \text{Tr}' \gamma_\mu \gamma_\rho \sigma_i \sigma_j S_0^2 R_{2v},
\end{aligned} \tag{40}$$

where $R_{2v} = 4(R_T + R_L)^2 + 8R_T(R_T - R_L)$. With the relative contribution of the longitudinal and transversal components of the gluon propagator it is possible to obtain simple relations between these two effective quark coupling constants. If it is assumed $\text{Tr}''(R_T^2 S_0^2) \gg \text{Tr}''(R_T R_L S_0^2)$ and $\gg \text{Tr}''(R_L^2 S_0^2)$ then:

$$\frac{g_{NJL}^{h.o.}}{g_{vNJL}} = 4 \frac{\text{Tr}'' S_0^2 [3R_T + R_L]^2}{\text{Tr}''(S_0^2 R_{2v})} \sim 3, \tag{41}$$

whereas for $\text{Tr}''(R_L^2 S_0^2) \gg \text{Tr}''(R_T R_L S_0^2)$ and $\gg \text{Tr}''(R_T^2 S_0^2)$ it yields:

$$\frac{g_{NJL}^{h.o.}}{g_{vNJL}} = 4 \frac{\text{Tr}''(S_0^2 [3R_T + R_L]^2)}{\text{Tr}''(S_0^2 [R_{2v}])} \sim 1. \tag{42}$$

These limits are in agreement with phenomenology [59]. The emerging quark-quark potential is therefore composed by several types of chiral invariant terms and this intricated structure is expected from a confining theory [31]. The expressions for sixth and eighth order effective quark interactions, found in Ref. [27], are reproduced without an explicit form of the gluon propagator. By rescaling these expressions according to the 't Hooft's large N_c scheme the n -quark effective interactions obtained from this expansion are seen to be of the order of N_c^{1-n} in agreement with the QCD large N_c expansion [43] although the present calculation includes a restricted class of diagrams for the model (1). There are certainly third and higher order couplings, 2-mesons-quark couplings and meson couplings to two quarks which will not be presented here. They are basically approximatedly one order of $1/m^*$ (or $1/(m^*)^2$) and also $1/N_c$ smaller than the second order axial pion-quark coupling shown above.

3.4 Pion sector

The longwavelength expansion provides pion self interactions in the lines of chiral perturbation theory. This was discussed with details in Refs.[6, 3, 4] for flavor SU(3). There are several contributions in the fourth order pion derivative and higher order in the pion field which also receives contributions from the derivative expansion. Since the main aim of the present work is to provide quark-pion couplings only the zero order derivative expansion terms will be shown here. This will not provide all the leading terms of chiral perturbation theory (up to the fourth order in pion momenta) and the missing term is obtained from the derivative expansion as shown below. The chirally symmetric third order pion self interactions are zero, in the zero order derivative expansion the fourth order pion self couplings from S_d are given by:

$$\begin{aligned}\mathcal{L}_4^{meson} &= -\frac{i}{4}Tr\{S_0\Phi_N\}^4 \\ &= -\frac{i}{4}Tr\{\gamma\cdot\mathcal{D}\pi_i(\gamma_5\sigma_i)\gamma\cdot\mathcal{D}\pi_k(\gamma_5\sigma_k)\gamma\cdot\mathcal{D}\pi_l(\gamma_5\sigma_l)\gamma\cdot\mathcal{D}\pi_m(\gamma_5\sigma_m)S_0^4\}.\end{aligned}\quad (43)$$

The traces in Dirac and flavor indices are given by:

$$\begin{aligned}tr(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) &= 4(g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho} - g^{\mu\rho}g^{\nu\sigma}), \\ tr(\sigma^i\sigma^j\sigma^k\sigma^l) &= 2(\delta^{ij}\delta^{kl} + \delta^{il}\delta^{jk} - \delta^{ik}\delta^{jl}).\end{aligned}\quad (44)$$

These fourth order terms $\mathcal{O}(N_c)$ can be written as:

$$\mathcal{L}_4^{meson} = -c_1(\mathcal{D}_\mu\pi_i\cdot\mathcal{D}^\mu\pi_i)^2 - c_2(\mathcal{D}_\mu\pi_i\cdot\mathcal{D}_\nu\pi_i)^2, \quad (45)$$

where $c_1 = -2c_2$ in agreement with Ref. [6] in the limit of point-mesons for which: $c_1 = 2iN_c Tr'' S_0^4$, where Tr'' stands for trace in spacetime or momentum indices. These are the most general chiral fourth order terms for single pion derivatives [8]. Although it is outside the scope of this work to present an extensive investigation of the resulting effective field theory, the leading symmetry breaking terms from this expansion are presented below in the zero order derivative expansion. Up to the second order in the current quark mass, the following symmetry breaking contributions appear:

$$\begin{aligned}\mathcal{L}_4^{sb} &= V_2 \frac{\vec{\pi}^4}{(1+\vec{\pi}^2)^2} + V_{3B} \frac{\vec{\pi}^2}{(1+\vec{\pi}^2)} (\mathcal{D}^\mu\vec{\pi}\cdot\mathcal{D}_\mu\vec{\pi}) - V_{3C} \frac{(\vec{\pi}\cdot\mathcal{D}^\mu\vec{\pi})^2}{(1+\vec{\pi}^2)} \\ &\quad + V_{4B} \frac{(\vec{\pi}^2)^2}{(1+\vec{\pi}^2)^2} (\mathcal{D}^\mu\vec{\pi}\cdot\mathcal{D}_\mu\vec{\pi}) + V_{4C} \frac{(\vec{\pi}\cdot\mathcal{D}^\mu\vec{\pi})(\vec{\pi}\cdot\mathcal{D}_\mu\vec{\pi})}{(1+\vec{\pi}^2)^2},\end{aligned}\quad (46)$$

where the following effective parameters (low energy coefficients) were defined:

$$V_2 = -iN_c 16 Tr'' m^2 S_0^2, \quad (47)$$

$$\begin{aligned}V_{3B} g^{\mu\nu} \delta_{ij} &= iN_c \frac{16}{3} Tr'' m \gamma_5 \gamma^\mu \gamma_5 \gamma^\nu \sigma_i \sigma_j S_0^3 \\ &\quad + V_{3C} g^{\mu\nu} \delta_{ij},\end{aligned}\quad (48)$$

$$V_{3C} g^{\mu\nu} \delta_{ij} = iN_c 4 Tr'' m \gamma^\mu \gamma^\nu \sigma_i \sigma_j S_0^3, \quad (49)$$

$$V_{4B} g^{\mu\nu} \delta_{ij} = -iN_c 4 Tr'' m^2 \gamma_5 \gamma^\mu \gamma_5 \gamma^\nu \sigma_i \sigma_j S_0^4, \quad (50)$$

$$V_{4C} g^{\mu\nu} = -iN_c 4 Tr'' m^2 \gamma_5 \gamma^\mu \gamma_5 \gamma^\nu S_0^4. \quad (51)$$

The next leading terms of the derivative expansion for the term V_2 contributes for other terms in I^{sb} .

3.4.1 Rewriting the pion sector

Let us rewrite the pion sector by considering the parameterization given in expression (15). For that, the derivative couplings must be extracted. The kernel S_0^N now is a function of the current quark mass M . The terms in the traces Tr can be written in terms of a part diagonal in coordinate space and another diagonal in momentum space [60], yielding the following form:

$$S_0 = \tilde{S}_0(i\gamma \cdot \partial + M), \quad (52)$$

where $\tilde{S}_0 = \tilde{S}_0(k) \equiv \frac{1}{k^2 - M^2}$. To write down all the chirally symmetric terms up to the fourth order the properties of the traces in flavor and Dirac matrices (44) were used. These terms up to the fourth order in the expansion of the determinant, always in the zero order derivative expansion, and up to the second order in the quark mass, for the chiral symmetry breaking but isospin invariant terms, are given by:

$$\begin{aligned} \mathcal{L}_\pi = & \frac{a_1}{2} tr \partial_\mu U \partial^\mu U^\dagger + a_{sb} tr M(U + U^\dagger) \\ & + l_1 (tr \partial_\mu U \partial^\mu U^\dagger)^2 + l_2 tr(\partial_\mu U^\dagger \partial_\nu U) tr(\partial^\mu U^\dagger \partial^\nu U) \\ & + l_3 tr [M(U^\dagger + U)] tr [\partial^\mu U \partial_\mu U^\dagger] \\ & + l_4 (tr [M(U^\dagger + U)])^2, \end{aligned} \quad (53)$$

where the trace tr stands for trace in flavor indices. By resolving Dirac and color indices traces, the low energy coefficients were defined as:

$$\begin{aligned} a_1 g_{\mu\nu} &= 2i^3 N_c Tr'' \tilde{S}_0^2 F^2 \gamma_\mu \gamma_\nu, \\ a_{sb} &= 4i N_c Tr'' \tilde{S}_0 F, \\ l_1 \Gamma_{\mu\nu\rho\sigma} &= -2 l_2 \Gamma_{\mu\nu\rho\sigma} = \frac{i^5}{2} N_c Tr'' \tilde{S}_0^4 F^4 \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma, \\ l_3 g_{\mu\nu} &= i N_c Tr'' \tilde{S}_0^3 F^3 \gamma_\mu \gamma_\nu, \\ l_4 &= -\frac{i}{2} N_c Tr'' \tilde{S}_0^2 F^2, \end{aligned} \quad (54)$$

where the traces in flavor were not included in Tr'' . However it is important to note that, as shown in Refs. [6, 4], by considering the leading terms of the derivative expansion with the full meson form factors, a different structure appears. In particular, for the following second order term of the determinant expansion $\partial_\mu U(x) \partial^\mu U^\dagger(y)$, that can be expanded in a derivative expansion. Consider a non trivial form factor $(G(p^2))$, non punctual pion) with which it can be written as:

$$\begin{aligned} & Tr \partial_\mu U(y) \partial^\mu U^\dagger(x) G(p^2) \\ \sim & Tr \left\{ \partial_\mu U(x) + (y-x)^\nu \partial_\nu \partial_\mu U(x) + \frac{1}{2} (y-x)^\rho (y-x)^\nu \partial_\rho \partial_\nu \partial_\mu U(x) \right\} \partial^\mu U^\dagger(x) G(p^2) \end{aligned} \quad (55)$$

where the first order term of this expansion is zero because $Tr (y - x)_\nu G(p^2) \sim i Tr \frac{\partial}{\partial p} G(p^2) \Big|_{p=0} = 0$. The second order term in expression (55) after an integration by parts can be written as:

$$I_2 = - tr_F G_{2,U} \partial^2 U \partial^2 U^\dagger \quad (56)$$

where, by resolving the trace as integration in momenta, $G_{2,U} = i2N_c \int_{p,q} \frac{\partial^2 G(p^2)}{\partial p^2}$. This term completes the leading terms of Chiral Perturbation Theory.

3.5 Complete second order effective model

Expressions (29,32,34,36,45,46) and the constituent quark and gluon free terms in expression (19), with the calculated corrections to the pion mass and quark effective mass (25), are written below together. With the field rescaling discussed above, the second order terms of the expansion it yields:

$$\begin{aligned} \mathcal{L}_{eff}^{(2)} = & t_1 \bar{\psi}_1 (i\gamma \cdot \partial - M^*) \psi_1 - 2t_1 g_A (\mathcal{D}_\mu \pi_i) \bar{\psi} i\gamma_5 \gamma^\mu \sigma_i \psi \\ & + t_1 g_{\pi d\pi} \frac{\vec{\pi} \times \partial^\nu \vec{\pi}}{1 + \vec{\pi}^2} \cdot (\bar{\psi} \gamma_\nu \vec{\sigma} \psi) + \frac{k_2}{2} \mathcal{D}_\mu \pi_i \mathcal{D}^\mu \pi_i \\ & - c_1 (\mathcal{D}_\mu \pi_i \cdot \mathcal{D}^\mu \pi_i)^2 - c_2 (\mathcal{D}_\mu \pi_i \cdot \mathcal{D}_\nu \pi_i)^2 \\ & + \mathcal{L}_{sb} + I_{h.o.d.} - t_1^2 \frac{g^2}{2} \int_y j_\mu^b(x) \tilde{R}_{bc}^{\mu\nu}(x, y) j_\nu^c(y) \\ & + t_1^2 \left\{ g_{NJL}^{h.o.} [(\bar{\psi}\psi)^2 + (\bar{\psi}\sigma_i \gamma_5 \psi)^2] + g_{vNJL} [(\bar{\psi}\sigma_i \gamma_\mu \psi)^2 + (\bar{\psi}\sigma_i \gamma_\mu \gamma_5 \psi)^2] \right\}, \end{aligned} \quad (57)$$

where \mathcal{L}_{sb} are the symmetry breaking terms up to the second order in the current quark mass, including (46),:

$$\begin{aligned} \mathcal{L}_{sb} = & c_{\sigma, sb} t_1 \frac{\vec{\pi}^2}{1 + \vec{\pi}^2} (\bar{\psi}\psi) - M_\pi^2 F^2 \frac{\vec{\pi}^2}{1 + \vec{\pi}^2} + V_2 \frac{\vec{\pi}^4}{(1 + \vec{\pi}^2)^2} \\ & + \frac{V_{3B} \vec{\pi}^2}{(1 + \vec{\pi}^2)} (\mathcal{D}^\mu \vec{\pi} \cdot \mathcal{D}_\mu \vec{\pi}) - V_{3C} \frac{(\vec{\pi} \cdot \mathcal{D}^\mu \vec{\pi})^2}{(1 + \vec{\pi}^2)} \\ & + \frac{V_{4B} (\vec{\pi}^2)^2}{(1 + \vec{\pi}^2)^2} (\mathcal{D}^\mu \vec{\pi} \cdot \mathcal{D}_\mu \vec{\pi}) + V_{4C} \frac{(\vec{\pi} \cdot \mathcal{D}^\mu \vec{\pi})(\vec{\pi} \cdot \mathcal{D}_\mu \vec{\pi})}{(1 + \vec{\pi}^2)^2}, \end{aligned} \quad (58)$$

$I_{h.o.d.}$ stands for higher order terms in momentum and in the derivative expansion. The (constituent) quark mass was defined as $M^* = m + \Delta m^*$, which is therefore not necessarily the same contribution as the effective mass m^* for the quarks that constitutes pions and the scalar condensate which can be determined from a gap equation. The terms with the effective parameters calculated above, such as Δm^* , g_A , $g_{\pi d\pi}$, M_π^2 , $c_{\sigma, sb}$, k_2 and V_i depend on the parameter t_2 , that was set equal to 1, and which indicates the quark component that was integrated out. The first three lines of expression (57) correspond to the large N_c constituent quark and pion effective theory proposed in Ref. [5] which however can receive corrections from the derivative expansion of the quark determinant. The effective quark-quark interactions and the symmetry breaking pion-quark coupling ($c_{\sigma, sb}$) and higher order pion interactions are of higher order in the $1/m^*$ (and also $1/N_c$) expansion.

4 Summary

In this work an effective model for quarks and pions was derived from the global color model (1). In the chiral limit, it corresponds to the large N_c effective theory proposed by Weinberg [5]. For that, the quark field was splitted into two components by means of the one loop background field method. The component that yields light quark-antiquark mesons and the chiral condensate was integrated out by means of the auxiliary field method. The one-loop level is obtained by performing the shift of all the quark bilinears which also is compatible with considering quark-antiquark light meson states. Therefore no criterium of the type of low and high energy modes was considered. However, it was noticed that this might involve an ambiguity in performing such splitting the quark field and this gives rise to a possible ambiguity in eventual contributions of the Skyrme terms and constituent (background) quarks to the baryon structure. Since the structure of the results are independent of the separation criterium, this is left for further investigations. This can be seen in expression (57) where the constituent quark contribution has the parameter t_1 whereas the Skyrme terms have implicitly $t_2^2 = 1$. The vector auxiliary fields that give rise to the local lightest vector meson fields were neglected because these excitations are considerably heavier than the pion and only contribute in relatively higher energies than the low energy regime dictated by pions. However, constituent quarks and pions are also expected to contribute for vector meson structure and dynamics and the analysis of the corresponding contribution is outside the scope of the work. Nevertheless it is interesting to remember that the fourth order (Skyrme) terms above, or corrections to them, might be obtained from the vector meson dynamics [50]. The part of this model that involves vector mesons interactions with quarks will be presented elsewhere. A chiral rotation has been performed for the limit of local structureless light mesons (pions) which yielded a particular dependence of the results on covariant derivatives of the quark and pion fields. Should the calculations be carried out without the structureless limit, some of the resulting expressions for the effective couplings or mass parameters would yield Schwinger-Dyson or Bethe-Salpeter like equations. Nevertheless results reproduce correctly the expected terms from Chiral Perturbation Theory for punctual pions, the expected leading pion-quark effective interactions as well as the GellMann Oakes Renner relation as the leading symmetry breaking low energy relation. A longwavelength and weak quark field expansion was performed by considering the local punctual meson fields in the zero order derivative expansion. The determinant expansion is also equivalent to a large quark effective mass m^* expansion or small coupling constant g^2 . For that, two different definitions of the pion field were considered which are related by chiral rotations from the original linear realization of chiral symmetry. In one of these pion definitions, the chiral invariant pion terms are always written in terms of the covariant pion derivative $(\mathcal{D}_\mu \vec{\pi})$ which allowed to compare the final expressions to the effective theory proposed in Ref. [5]. With the redefinition of the pion and quark fields a non trivial Jacobian appears in the functional measure of the generating functional which was not exhibited and discussed in this work because it involves higher order quark-pion interactions, of the form $\mathcal{O}(\bar{\psi}\zeta\psi(\vec{\pi})^n)$. The pion sector was rewritten in terms of a second pion definition showing that it turns out to provide basically the leading terms of chiral perturbation theory. The corresponding expressions for the low energy constants were exhibited. The effective coupling constants and parameters of the effective model (57) were expressed in

terms of the coupling g^2 and quark current and constituent, m^* , masses. All the derivation was carried out without specifying the gluon propagator what would be needed to provide numerical estimations. The present approach makes possible to introduce further systematic corrections in the expansions that were performed. By closing external legs the higher order terms of the expansion yield loop corrections for the effective couplings and parameters above with progressively large powers of $1/m^*$. Corrections for intermediary, or relatively larger, energies might also be considered in the derivative expansion and by considering the full meson form factors. Finally, higher order corrections to the background field method can also be envisaged.

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